Digital Systems

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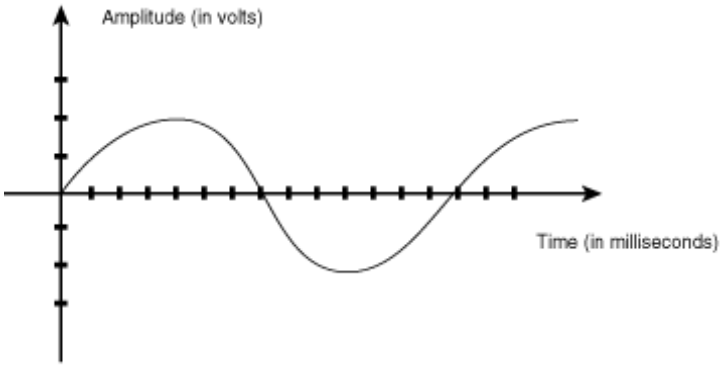
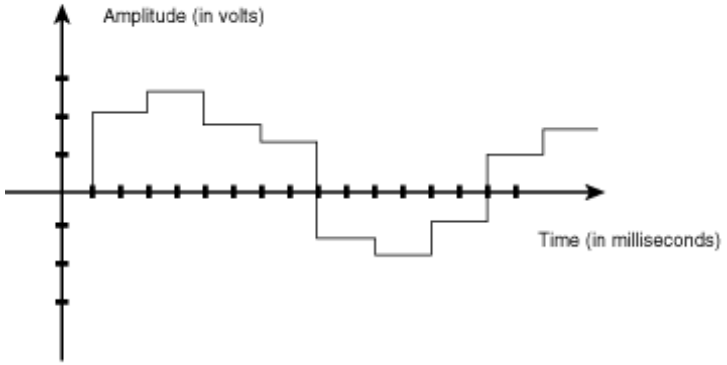
Introduction

The advent of the digital age

- Analog vs. digital?



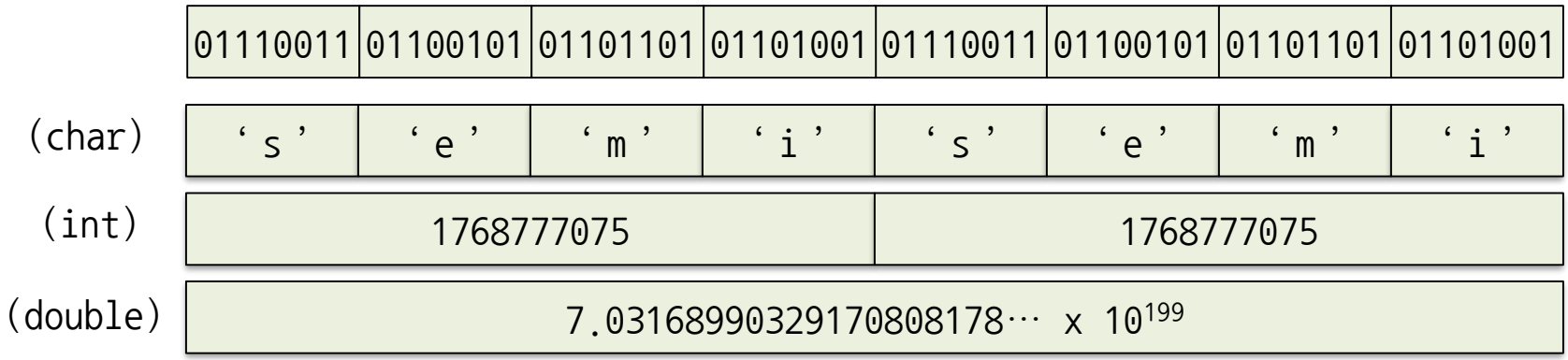


- Compact disc (CD)
  - 44.1 KHz, 16-bit, 2-channel
- MP3
  - A digital audio encoding with lossy data compression

Representing Information

Information = Bits + Context

- Computers manipulate representations of things
- Things are represented as binary digits
- What can you represent with N bits?
  - 2^N things
  - Numbers, characters, pixels, positions, source code, executable files, machine instructions, …
  - Depends on what operations you do on them

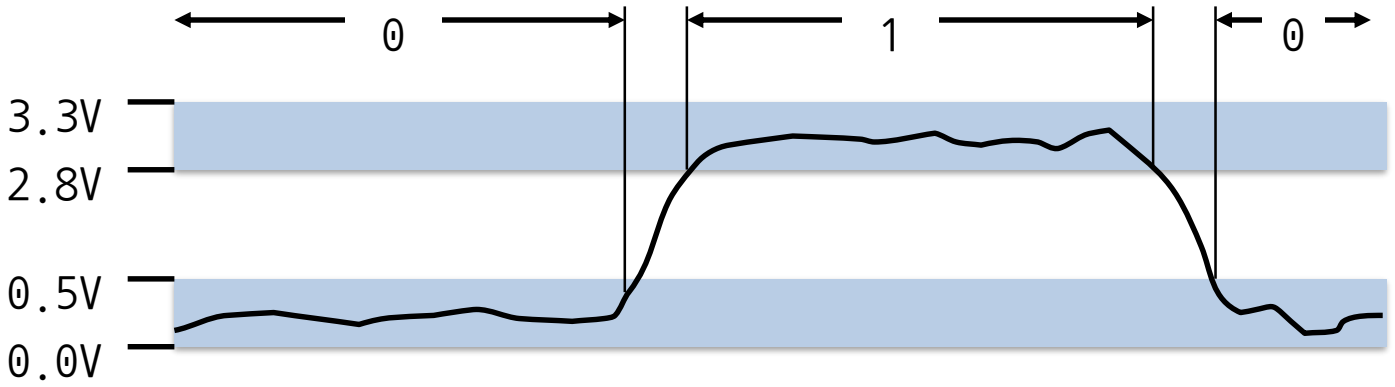


Binary Representations

Why not base 10 representation?

- Easy to store with bistable elements
- Straightforward implementation of arithmetic functions
- Reliably transmitted on noisy and inaccurate wires

Electronic implementation



Encoding Byte Values

Byte = 8 bits

- Binary: **00000000_2** to **11111111_2**
- Octal: **000_8** to **377_8**
  - An integer constant that begins with 0 is an octal number in C
- Decimal: **0_{10}** to **255_{10}**
  - First digit must not be 0 in C
- Hexadecimal: **00_{16}** to **FF_{16}**
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write **$FA1D37B_{16}$** in C as **0xFA1D37B** or **0xfa1d37b**

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Boolean Algebra (1)

Developed by George Boole in 1849

- Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

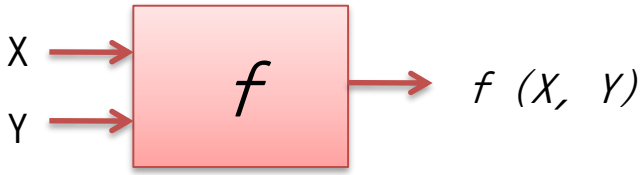
Exclusive-Or (Xor)

- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Boolean Algebra (2)

0	0	1	1	X
0	1	0	1	Y
0	0	0	0	Constant 0
0	0	0	1	X & Y ; AND
0	0	1	0	~ (X → Y)
0	0	1	1	X
0	1	0	0	~ (Y → X)
0	1	0	1	Y
0	1	1	0	X ^ Y ; XOR
0	1	1	1	X ¦ Y ; OR
1	0	0	0	~ (X ¦ Y) ; NOR
1	0	0	1	~ (X ^ Y) ; X-NOR
1	0	1	0	~ Y
1	0	1	1	Y → X
1	1	0	0	~ X
1	1	0	1	X → Y
1	1	1	0	~ (X & Y) ; NAND
1	1	1	1	Constant 1

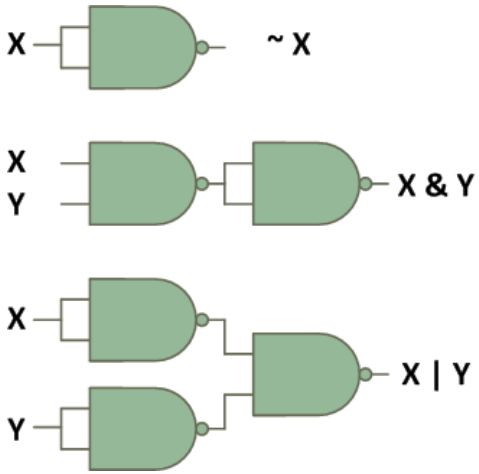


Basic operations: AND(&), OR(¦), NOT(~)

X ^ Y = (X & ~Y) ¦ (~X & Y)

X → Y = ~X ¦ Y

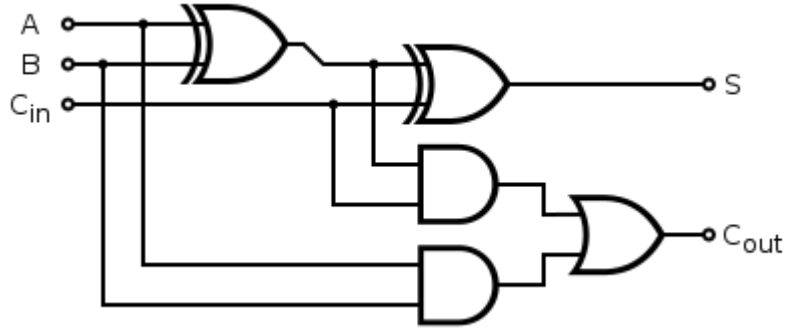
A complete set: NAND = ~ (X & Y)

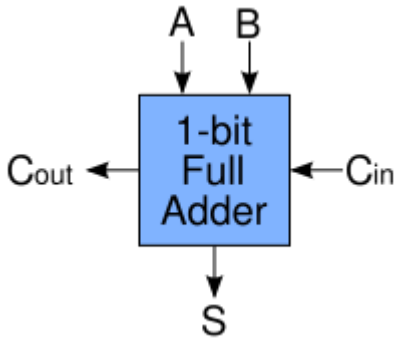


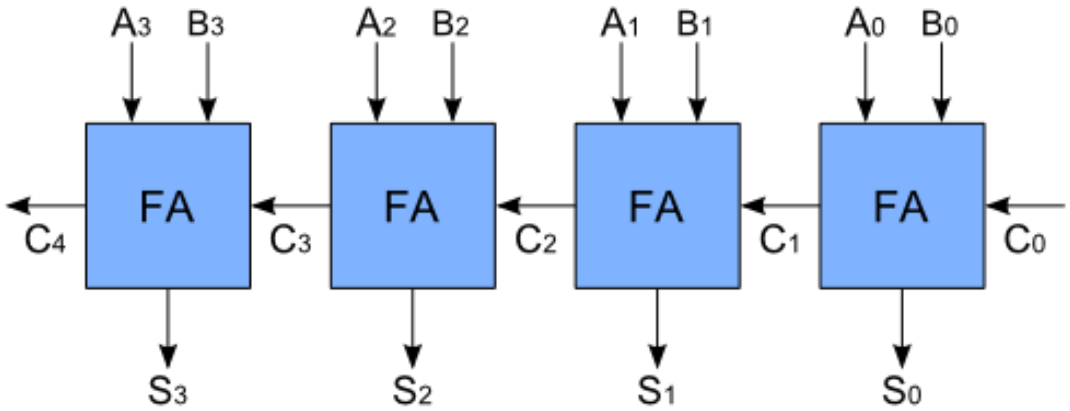
Combinational Logic

Adder

Full Adder



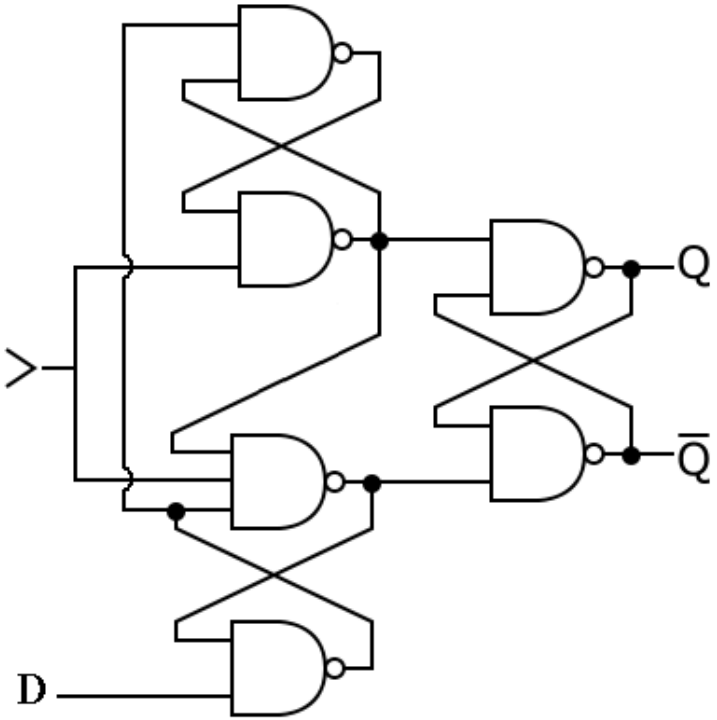




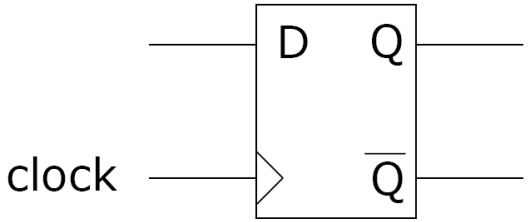
4-bit Ripple Carry Adder

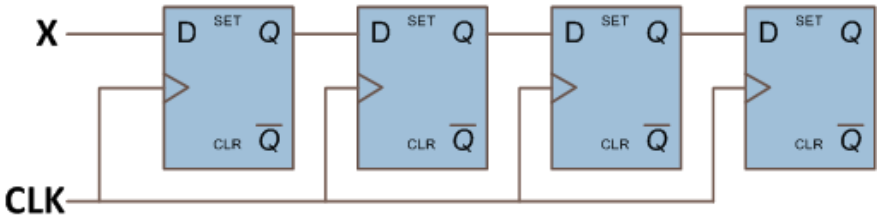
Sequential Logic

Flip-flops

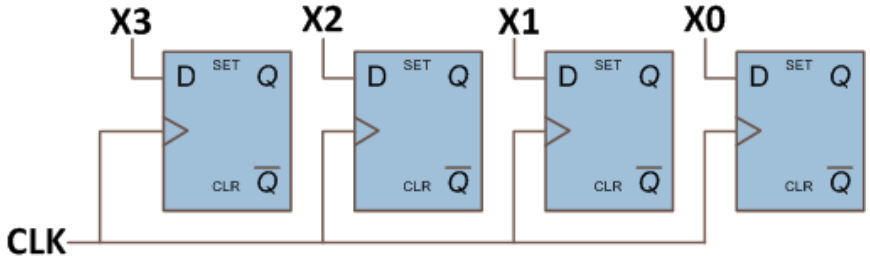


Edge triggered D flip-flop





Shifter



4-bit register

Digital Systems

Summary

- Boolean algebra is a mathematical foundation for modern digital systems
- Boolean algebra provides an effective means of describing circuits built with switches
  - Claude Shannon in the late 1930's
- You can build any digital systems with NAND gates
- A NAND gate can be easily built with CMOS transistors
- The transistor is the basic building block for digital systems

Intel Xeon 7560 (8-core): 2.3B transistors